Response to Referee A

The manuscript reports Direct Numerical Simulations of the compressible magnetohydrodynamic equations. According to the authors, the main finding is that the small-scale dynamo phase is followed by a large-scale dynamo phase. The authors call the large-scale growth ”quasi-kinematic”, although there is little evidence of that (and no proper definition of this term). The paper is poorly written and lacks precise and clear evidence of the results that are advertised. Here are more detailed comments:

We thank the referee for carefully reading and reviewing our manuscript. We regret that the referee has found the paper poorly written and also is not satisfied with the presentation of our results. We have revised certain parts of the paper, especially the introductory paragraphs. Here we offer explanations to the referee’s detailed comments and have pointed to the revisions (marked in blue) pertaining to some of the comments. Though we had reasoned the naming of the large-scale dynamo as ’quasi-kinematic’ in the discussions, we now have revised to include a definition of the term ’quasi-kinematic’ upfront in the introductory paragraphs itself. We hope that these revisions will satisfy the referee to find our manuscript now suitable for a publication in PRL.

1. Why use compressible equations to describe effects that seem completely independent of compressibility?

In the incompressible case, the pressure depends nonlocally on all points in the domain. This requires global communication between two time steps, which is inefficient and avoided by our approach. There is no significant difference in the results between the two cases. We have now added a reference to that extent.

2. In the bottom panel of figures 1 and 2, it is incorrect to say that three stages can be clearly identified. In figure 2, the second stage does not correspond to a plateau, and starting from t=50 the curve looks like a decaying exponential.

We agree with the reviewer that in Fig 2. (of the paper), the transition to the second stage is not very sharp. Please note that the difference between the runs in Figs 1 and 2 (of the paper) is that the former has uniform shear as an ingredient in the dynamo action, which is absent in the latter. We believe that this is responsible for the more sharp transition in the former case. The presence of 3 stages thus is much clearer for the case with shear in Fig 1 (of the paper). However, a second stage does exist for Run B (without shear) in the Fig 2 (of the paper) also. The plateaus in Figs. 1 and 2 of the paper do have some fluctuations. But as the system is highly turbulent, it is natural to expect such fluctuations. Well, it is instructive to compare with cases where we know for a fact that a second phase of growth is not expected. Below in Figure 1, we compare the growth rate of the $M_1$ mode from Run B (as in Fig. 2 of the paper) with the growth rate of $M_1$ from run D (purely small-scale dynamo case) in the left panel. In the right panel, the comparison is between growth rates of $M_1$ and $M_4$ from Run B itself. From both these panels, the fluctuating nature of the growth rate curves can be gathered. In fact the curve from Run D shown in comparison even dips far below $\gamma = 0$ line, thus being consistent with no growth after the initial kinematic phase. Thus, it is easier to see that in comparison, the $M_1$ growth rate curve has a definitive second plateau around $t = 120$ to $t = 250$. In view of the referee’s query, we have now added the left panel of this figure also as an inset into Fig. 2 of our revised paper. Further, we have added such an inset also in Fig. 1.

3. The authors associate their ”quasi-kinematic” dynamo to an exponential growth of the large-scale field, together with a linear increase of the magnetic intensity at the
forcing scale (M4). However, this contradicts the theory of large-scale dynamos: in standard alpha effect theory, the small-scale field is proportional to the large-scale one and both grow exponentially.

The interpretation of standard mean field theory has been problematic for quite a long time. The dynamo community for long has expected large-scale field growth right in the kinematic phase. We show both in this paper and our previous work [1], that this is not correct due to the presence of fast growing small-scale fields. The kinematic phase is largely governed by the small-scale dynamo (SSD) in the large-Re\_M regime, with hints of the large-scale field growth shown in the polarization data. The large-scale field is seen to be negatively polarized due to the helical turbulence as opposed to the positive polarization of the small-scale fields [1]. Only when Re\_M is small (such that SSD is not super-critical), a kinematic large-scale dynamo can be expected [2]. In fact, the standard mean-field theory predicts a large-scale dynamo only when the small-scale fields are produced solely via tangling of the large-scale fields [3]. In large-Re\_M systems, the main producer of small-scale fields is the SSD which has traditionally been unaccounted for in the standard mean-field theory.

In this work, we show that only when the small-scale dynamo slows down that the large-scale field grows at the rate governed solely by the large-scale dynamo. This system may or may not be fully compatible with the standard mean-field theory (MFT), which we have only begun to explore. While we find that the growth rates of the large-scale dynamo seem to agree with estimates with standard MFT, we don’t think it is the full story yet. We understand that the referee’s expectations were also set by our claim that the quasi-kinematic LSD matches with the standard LSD. However, this claim was only from the point of view that the results indicate that the standard theory maybe approximately correct. To give a more accurate picture, we now say that quasi-kinematic LSD is only similar to the standard LSD (marked in blue on page 2). As the referee has rightly pointed out, here the large-scale and small-scale fields do not grow at the same rate, as an ”eigenfunction”. This is because, in this phase, small-scale fields are produced by both the tangling of the large-scale field and also by the nonlinear small-scale dynamo. Thus its evolution can be more complex and will not mirror the large scale field. But there would still be a part of it correlated with large-scale fields to produce a mean-emf which drives the large scale dynamo.

Figure 1: shows \( \gamma \), the growth rate of \( M_1 \).
In the discussions, we have mentioned that the system is already nonlinear before the second phase of large-scale field growth arises. The standard mean-field equation equations/solutions do not account for this nonlinearity. However, the reason we think these estimates agree is that the field at large-scale field is not affected by Lorentz force yet.

We think our result is an important one which shows that indeed standard MFT needs rethinking. However, we agree that our discussions may have emphasized on the agreement of the numerics with the theory too much. We have revised the relevant line in the conclusions in our last paragraph (marked in blue).

4. When there is shear, the magnitude of the alpha effect coefficient typically depends on the shear intensity.

Well, in our case, the shear parameter is small enough to make the shear time scale much larger than the turbulent correlation time scale. We had mentioned on page 2, the third line in the paragraph after the Eqs. (1) and (2) - ”We have checked that the shear in our simulations is small enough that its effect on SSD growth is unimportant.” This is the case, when the shear parameter $S\tau$ is small, where $\tau$ is the correlation time of the turbulence. Thus, related anisotropy effects are expected to be small also. Our estimates are calculated using the simplest model and it can be improved, but that is beyond the scope of the present paper where the focus is on the important numerical result of quasi-kinematic large-scale dynamo.

We have changed the line to explicitly mention that our shear parameter is small enough for shearing time scale to be larger than the eddy turn over time scale (marked in blue on page 2).

5. There is some inconsistency between the theoretical estimates and the numerical values of the growth rate, but instead of discussing them the inconsistencies are hidden in some ”efficiency factor”

While our referee has noted that the numerical results are off from the theoretical estimates, it is to be appreciated that they are off by a factor that is uniform across the runs with different values of the shear parameter (while all other parameters are the same) and is of order unity. However, we agree with the referee that we did not discuss the effects which have not been included. We do so now and have included a couple of lines in the main text. These have been marked in blue on page 4.

To conclude, the results advertised in the title and abstract are not present in the numerical data (no well defined plateaus) and the interpretation is both qualitatively and quantitatively inconsistent with the well-established large-scale dynamo theory. For these reasons I cannot recommend this paper for publication in PRL.

We emphasize that the standard large-scale dynamo theory does not account for the effects of rapidly growing fluctuations due to the small scale dynamo. However, our direct simulations have clearly shown that a distinct large-scale dynamo does operate when the small-scale dynamo begins to saturate. This is apparent from the second plateau in the growth rate, albeit one which as we have demonstrated is naturally afflicted by minor fluctuations. As we have explained, we are in novel territory where the standard large-scale dynamo theory actually needs revision.

We hope that our explanations convince the reviewer that this result of quasi-kinematic large-scale field growth is of high importance and our study is only a preliminary one but nonetheless that which warrants the best exposure to the rest of the community.
References

